Tell me everything you know about rational functions

(hint: in case you don't remember, rational functions are quotients with a polynomial in the numerator and denominator like -)

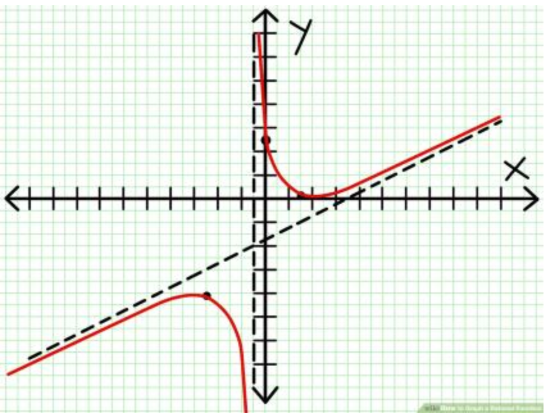
Unit 1 Day 7 (1.7):
Rational Function End Behavior

Quiz Today!

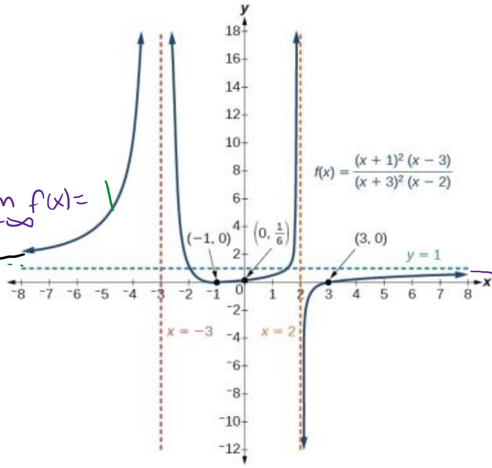
Homework:
Rational Function End Behavior

Rational Functions

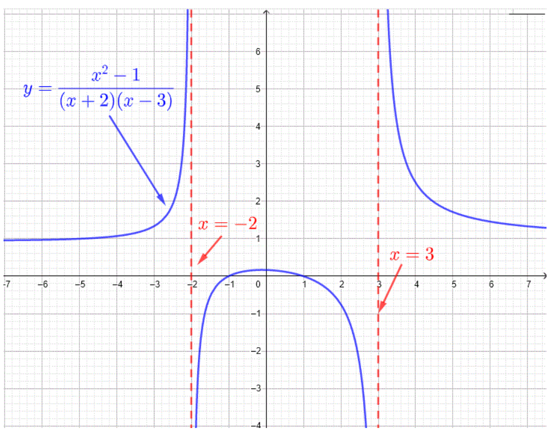
A rational function is a function that can be written as the quotient of two polynomial functions.
The function $f(x) = \frac{n(x)}{d(x)}$ is rational if $n(x)$ and $d(x)$ are polynomials

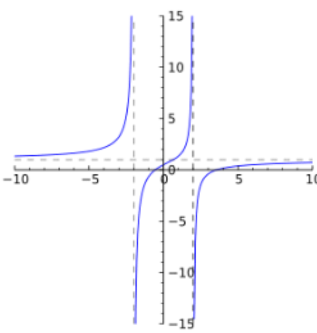


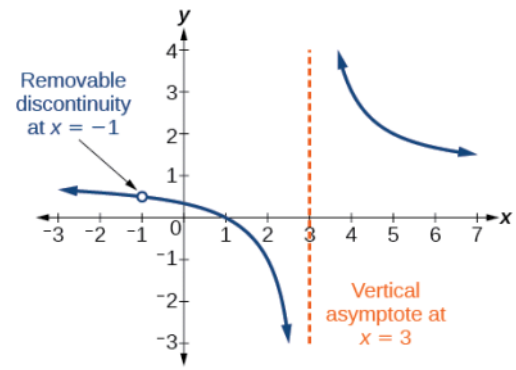
$\lim_{x \to -\infty} f(x) = 1$



$\lim_{x \to \infty} f(x) = 1$







Finding Horizontal Asymptotes

To find the horizontal asymptote, compare the degree of the numerator and denominator. Three cases arise:

Case	Degree		Degree	Asymptote
1	Numerator	$<$	Denominator	$y=0$
2	Numerator	$>$	Denominator	
3	Numerator	$=$	Denominator	$y=$ divide coefficients

Rational functions can have at most ONE HA

y=

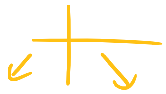
BOBO BOTN EATS DC

one

BOTN

$$f(x)=\frac{5x^2+10x-6x^4+7}{5+x+2x^2} \qquad g(x)=\frac{x^4-16}{2+3x} \qquad h(x)=\frac{3-6x+10x^2}{4-5x}$$

$$\frac{-6x^4}{2x^2} = -3x^2$$



$$\frac{x^4}{3x^1} = \frac{1}{3}x^3$$

$$\frac{10x^2}{-5x} = -2x$$

$\lim_{x\to-\infty} f(x) = -\infty$

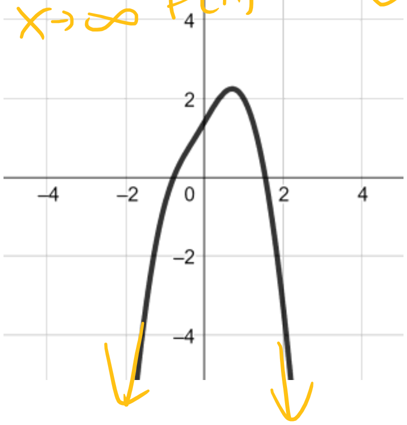
$\lim_{x\to\infty} f(x) = -\infty$

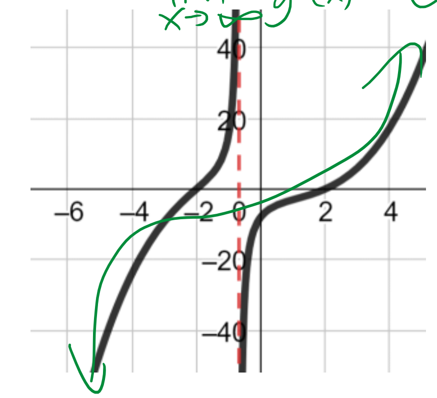
$\lim_{x\to-\infty} g(x) = -\infty$

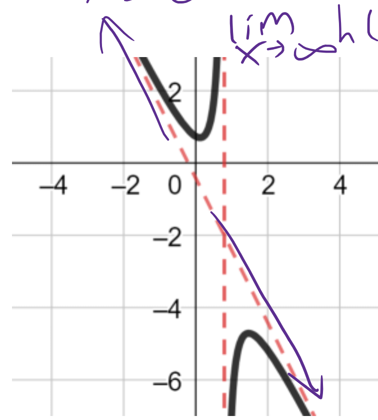
$\lim_{x\to\infty} g(x) = \infty$

$\lim_{x\to-\infty} h(x) = \infty$

$\lim_{x\to\infty} h(x) = -\infty$







EATS DC

$$f(x) = \frac{5x^2 + 10x - 6x^3 + 7}{5 + x + 2x^3}$$

HA: $y = -3$

$\lim_{x \to -\infty} f(x) = -3$ $\lim_{x \to \infty} f(x) = -3$

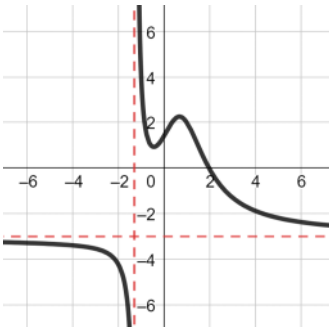
$$g(x) = \frac{16x - 1}{2 + 4x}$$

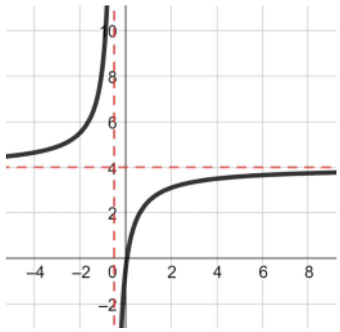
HA: $y = 4$

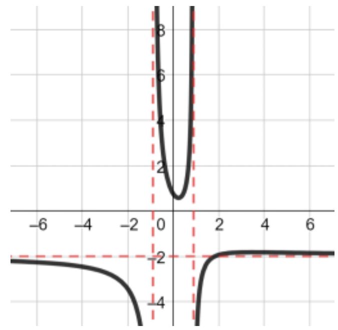
$\lim_{x \to -\infty} g(x) = 4$ $\lim_{x \to \infty} g(x) = 4$

$$h(x) = \frac{3 - 6x + 10x^2}{4 - 5x^2}$$

$\lim_{x \to -\infty} h(x) = -2$ $\lim_{x \to \infty} h(x) = -2$







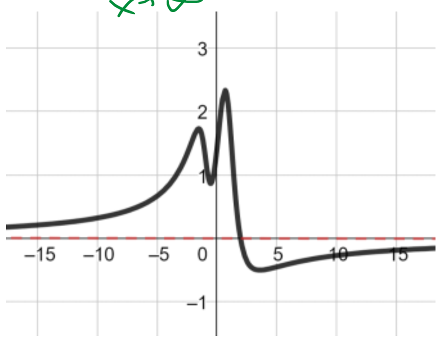
BOBO

$$f(x) = \frac{5x^2 + 10x - 6x^3 + 7}{5 + x + 2x^4}$$

HA: $y=0$

$\lim_{x \to -\infty} f(x) = 0$

$\lim_{x \to \infty} f(x) = 0$

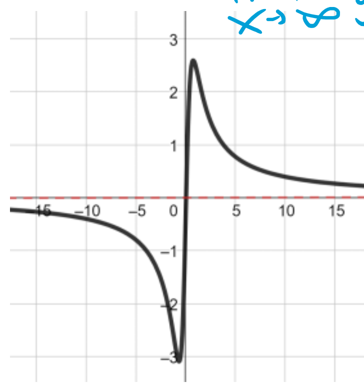


$$g(x) = \frac{16x - 1}{2 + 4x^2}$$

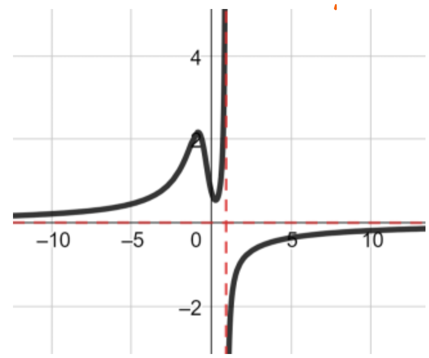
HA: $y=0$

$\lim_{x \to -\infty} g(x) = 0$

$\lim_{x \to \infty} g(x) = 0$



$$h(x) = \frac{3 - 6x + 10x^2}{4 - 5x^3}$$



Find the equation of the horizontal asymptote, if there is one. Describe the end behavior using limit notation.

A) $f(x) = \frac{16x+1}{4x^2-2}$

HA: $y=0$

$\lim_{x \to -\infty} f(x) = 0$

$\lim_{x \to \infty} f(x) = 0$

B) $g(x) = \frac{16x^2+1}{4x^2-2}$

HA: $y=4$

$\lim_{x \to -\infty} g(x) = 4$

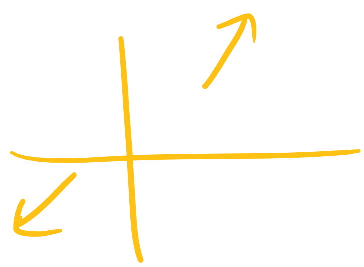
$\lim_{x \to \infty} g(x) = 4$

C) $h(x) = \frac{16x^3+1}{4x^2-2}$

$\frac{16x^3}{4x^2} = 4x$

$\lim_{x \to -\infty} h(x) = -\infty$

$\lim_{x \to \infty} h(x) = \infty$



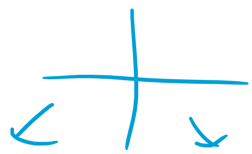
Write the equation of a rational function with the following properties

A)

- $\lim_{x \to -\infty} f(x) = -\infty$
- $\lim_{x \to \infty} f(x) = -\infty$
- Degree of Denominator: 2

$$f(x) = \frac{-3x^4 - 5x}{3x^2 + 6x}$$

$$\frac{-3x^4}{3x^2} = -x^2$$



B)

- Has a slant asymptote
- Slant asymptote has a slope of -3
- The leading coefficient of the polynomial in the numerator is 6

$$\frac{6x^3 + 4}{-2x^2 + 3}$$

$$\frac{6x^3}{-2x^2} = -3x$$